MODEL QUESTION PAPER

MATHEMATICS

XII-STANDARD (STATE BOARD)

PART-I **Maximum Marks: 90**

All Questions are **compulsory.** **20 x 1 = 20**

1	If $\lvert adj\,(adj\,A)\rvert = \lvert A\rvert^9$, then the order of the square matrix A is 1) 3 2) 4 3) 2 4) 5	1
2	If $A = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$, then adj (adj A) is 1) $\begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$ 2) $\begin{pmatrix} 6 & -6 & 8 \\ 4 & -6 & 8 \\ 0 & -2 & 2 \end{pmatrix}$ 3) $\begin{pmatrix} -3 & 3 & -4 \\ -2 & 3 & -4 \\ 0 & 1 & -1 \end{pmatrix}$ 4) $\begin{pmatrix} 3 & -3 & 4 \\ 0 & -1 & 1 \\ 2 & -3 & 4 \end{pmatrix}$	1
3	$i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is 1) 0 2) 1 3) -1 4) i	1
4	If $\omega = cis\,\frac{2\pi}{3}$, then the number of distinct roots of $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$ 1)1 2) 2 3) 3 4) 4	1
5	A zero of $x^3 + 64$ is 1) 0 2) 4 3) 4i 4) -4	1
6	The value of $\sin^{-1}(\cos x), 0 \le x \le \pi$ is 1) $\pi - x$ 2) $x - \frac{\pi}{2}$ 3) $\frac{\pi}{2} - x$ 4) $x - \pi$	1

7	The equation of the circle passing through (1,5) and (4,1) and touching y – axis is $x^2+y^2-5x-6y+9+\lambda(4x+3y-19)=0$ where λ is equal to 1) $0, \frac{4}{9}$ 2) 0 3) $\frac{40}{9}$ 4) $\frac{-40}{9}$	1
8	The values of m for which the line $y=mx+2\sqrt{5}$ touches the hyperbola $16x^2-9y^2=144$ are the roots of $x^2-(a+b)x-4=0$, then the value of $(a+b)$ is 1) 2 2) 4 3) 0 4) -2	1
9	If $\vec{a}$ and $\vec{b}$ are parallel vectors, then $\left[\vec{a}, \vec{c}, \vec{b}\right]$ is equal to 1) 2 2) -1 3) 1 4) 0	1
10	The volume of a sphere is increasing in volume at the rate of $3\pi cm^3/sec$ The rate of change of its radius when radius is $\frac{1}{2}$ cm (1) 3 cm/s (2) 2 cm/s (3) 1 cm/s (4) $\frac{1}{2}$ cm/s	1
11	The point of inflection of the curve $y=(x-1)^3$ is (1) (0,0) (2) (0,1) (3) (1,0) (4) (1,1)	1
12	A circular template has a radius of 10 cm. The measurement of radius has an approximate error of 0.02 cm. Then the percentage error in calculating area of this template is (1) 0.2% (2) 0.4% (3) 0.04% (4) 0.08%	1
13	The value of $\int_0^{\frac{2}{3}} \frac{dx}{\sqrt{4-9x^2}}$ is 1) $\frac{\pi}{6}$ 2b) $\frac{\pi}{2}$ 3) $\frac{\pi}{4}$ 4) π	1
14	If $\int_0^x f(t)dt = x + \int_x^{-1} tf(t)dt$, , then the value of f(1) is 1) $\frac{1}{2}$ 2) 2 3) 1 4) $\frac{3}{4}$	1

15	The slope at any point of a curve $y = f(x)$ is given by $\frac{dy}{dx} = 3x^2$ and it passes through (-1,1). Then the equation of the curve is 1) $y = x^3 + 2$ 2) $y = 3x^2 + 4$ 3) $y = 3x^3 + 4$ 4) $y = x^3 + 5$	1
16	The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{3}} + x^{\frac{1}{4}} = 0$ are respectively 1) 2, 3 2) 3, 3 3) 2, 6 4) 2, 4	1
17	Let X be random variable with probability density function $f(x) = \begin{cases} \frac{\ }{x^3} & x \geq 1 \\ 0 & x < 1 \end{cases}$ Which of the following statement is correct 1) both mean and variance exist 2) mean exists but variance does not exist 3) both mean and variance do not exist 4) variance exists but Mean does not exist.	1
18	If the length of the perpendicular from the origin to the plane $2x + 3y + \lambda z = 1, \lambda > 0$ is $\frac{1}{5}$, then the value of λ is 1) $2\sqrt{3}$ 2) $3\sqrt{2}$ 3) 0 4) 1	1
19	A computer salesperson knows from his past experience that he sells computers to one in every twenty customers who enter the showroom. What is the probability that he will sell a computer to exactly two of the next three customers? 1) $\frac{57}{20^3}$ 2) $\frac{57}{20^2}$ 3) $\frac{19^3}{20^3}$ 4) $\frac{57}{20}$	1
20	Which one of the following is not true? 1) Negation of a negation of a statement is the statement itself. 2) If the last column of the truth table contains only *T* then it is a tautology. 3) If the last column of its truth table contains only *F* then it is a contradiction 4) If *p* and *q* are any two statements then $p \leftrightarrow q$ is a tautology.	1

PART-II

Answer any **seven questions**. Question No. **30** is **compulsory** **7 x 2 = 14**

21	Solve the following system of linear equations, using matrix inversion method: $5x + 2y = 3$, $3x + 2y = 5$.	2
22	Obtain the Cartesian equation for the locus of $z = x + iy$ in each of the following cases: $\|z - 4\| = 16$	2
23	Find a polynomial equation of minimum degree with rational coefficients, having $2i+3$ as a root.	2
24	The parabolic communication antenna has a focus at $2m$ distance from the vertex of the antenna. Find the width of the antenna $3m$ from the vertex.	2
25	Find the angle between the line $\vec{r}=\left(2\hat{i}-\hat{j}+\hat{k}\right)+t\left(\hat{i}+2\hat{j}-2\hat{k}\right)$ and the plane $\vec{r}.\left(6\hat{i}+3\hat{j}+2\hat{k}\right)=8$.	2
26	Find two positive numbers whose sum is 12 and their product is maximum.	2
27	Assuming $\log_{10} e = 0.4343$ find an approximate value of $\log_{10} 1003$.	2
28	Evaluate $\int_0^1 x\,dx$ as the limit of a sum.	2
29	Three fair coins are tossed simultaneously. Find the probability mass function for number of heads occurred.	2
30	Establish the equivalence property connecting the bi-conditional with conditional: $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$	2

PART-III

Answer any **seven questions.** Question No. **40** is **compulsory** **7 x 3 = 21**

31	If A = $\begin{pmatrix} 4 & 3 \\ 2 & 5 \end{pmatrix}$ find x and y such that $A^2 + xA + yI_2 = O_2$. Hence, find A^{-1}.	3
32	Find the square roots of 4 + 3i.	3
33	Solve the cubic equation : $2x^3 - x^2 - 18x + 9 = 0$ if sum of two of its roots vanishes.	3
34	Solve $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1} x$ for $x > 0$	3
35	Find the torque of the resultant of the three forces represented by $-3\hat{i}+6\hat{j}-3\hat{k}, 4\hat{i}-10\hat{j}+12\hat{k}$ and $4\hat{i}+7\hat{j}$ acting at the point with position vector $8\hat{i}-6\hat{j}-4\hat{k}$, about the point with position vector $18\hat{i}+3\hat{j}-9\hat{k}$.	3
36	Using the l'Hôpital Rule prove that, $\lim_{x\to 0^+} (1+x)^{\frac{1}{x}} = e$	3
37	Find the partial derivatives of the following functions at the indicated points $h(x,y,z) = x\sin(xy) + z^2x, \left(2, \frac{\pi}{4}, 1\right)$	3
38	The time to failure in thousands of hours of an electronic equipment used in a manufactured computer has the density function $f(x) = \begin{cases} 3e^{-3x}, & x > 30 \\ 0, & elsewhere \end{cases}$. Find the expected life of this electronic equipment.	3
39	Let $*$ be defined on **R** by $(a*b) = a+b+ab-7$. Is * binary on **R**? If so, find $3*\left(\frac{-7}{15}\right)$	3

40	Solve : $\frac{dy}{dx}+2y\cot x=3x^2\operatorname{cosec}^2 x$.	3

PART-IV

Answer all the **questions .** **7 x 5 = 35**

41	a) Investigate for what values of λ and μ the system of linear equations. $x + 2y + z = 7$, $x + y + \lambda z = \mu$, $x + 3y - 5z = 5$. **(OR)** b) If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right)=\frac{\pi}{4}$, then show that $x^2 + y^2 + 3x - 3y + 2 = 0$.	5
42	a) Solve the equation $6x^4-5x^3-38x^2-5x+6=0$ if it is known that $\frac{1}{3}$ is a solution. **(OR)** b) Solve : $\cot^{-1}x-\cot^{-1}(x+2)=\frac{\pi}{12}, x>0$	5
43	a) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of $4m$ when it is $6m$ away from the point of projection. Finally it reaches the ground $12m$ away from the starting point. Find the angle of projection. **(OR)** b) Prove by vector method that $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$	5
44	a) Find the parametric vector, non-parametric vector and Cartesian form of the equations of the plane passing through the points (3, 6, -2), (-1, -2, 6), and (6, -4, -2). **(OR)** b) Find the acute angle between $y=x^2$ and $y=(x-3)^2$.	5
45	a) Sketch the graphs of the following functions: $y=-\frac{1}{3}(x^3-3x+2)$ **(OR)** b) $w(x,y,z)=xy+yz+zx$, $x=u-v, y=uv, z=u+v, u,v\in R$. Find $\frac{\partial w}{\partial s}$, $\frac{\partial w}{\partial v}$ and evaluate them at $\left(\frac{1}{2},1\right)$	5

46 a) Find the area of the region common to the circle $x^2+y^2=16$ and the parabola $y^2=6x$ **5**

(OR)

b) The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that the number triples in 5 hours, find how many bacteria will be present after 10 hours?

47 a) A random variable X has the following probability mass function. **5**

x	1	2	3	4	5	6
f(x)	k	2k	6k	5k	6k	10k

Find (i) $P(2<X<6)$ (ii) $P(2\le X<5)$ (iii) $P(X\le 4)$ (iv) $P(3<X)$

(OR)

b) Let A= $A=\begin{pmatrix}1&0&1&0\\0&1&0&1\\1&0&0&1\end{pmatrix}$, $B=\begin{pmatrix}0&1&0&1\\1&0&1&0\\1&0&0&1\end{pmatrix}$ $C=\begin{pmatrix}1&1&0&1\\0&1&1&0\\1&1&1&1\end{pmatrix}$ be any three boolean matrices of the same type. Find (i) $A\vee B$ (ii) $A\wedge B$ (iii) $(A\vee B)\wedge C$ (iv) $(A\wedge B)\vee C$.