MODEL QUESTION PAPER

MATHEMATICS

XII-STANDARD (STATE BOARD)

Time : 3.00 hrs **Max Marks : 90**

PART I

i) All questions are **compulsory.** **20 X 1 = 20**

ii) Choose the most appropriate answer from the given **four** alternatives and write the answer along with the code**.**

S.No.	Questions	Marks
1	If $A[1 \ -2 \ 1 \ \ 4] = [6 \ 0 \ 0 \ 6]$, then $A =$ (1)$[1 \ -2 \ 1 \ \ 4]$ (2)$[\ 1 \ 2 \ -1 \ 4]$ (3)$[\ 4 \ 2 \ -1 \ 1]$ (4)$[4 \ -1 \ 2 \ \ 1]$	**1**
2	If Crammar's rule can be applied then (1) $\Delta\neq0$ (2) $\Delta=0$ (3) $\Delta_X = 0$ (4) $\Delta_X \neq 0$	**1**
3	Find *the value of* $\sum_{n=1}^{13}(i^n + i^{n-1})$ (1)$1 + i$ (2)i (3)1 (4)0	**1**
4	A zero of $x^3 + 64$ is (1)0 (2)4 (3)$4i$ (4)-4	**1**
5	If $x \ + y \ = \frac{2\pi}{3}$; Then $x \ + y$ is equal to (1)$\frac{2\pi}{3}$ (2)$\frac{\pi}{3}$ (3)$\frac{\pi}{6}$ (4)π	**1**
6	Tangents are drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ parallel to the straight line $2x - y = 1$. One of the points of contact of tangents on the hyperbola is (1)$\left(\frac{9}{2\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ (2)$\left(\frac{-9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (3)$\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (4)$\left(3\sqrt{3}, -2\sqrt{2}\right)$	**1**
7	*The angle between two lines* $\frac{x-2}{3} = \frac{y+1}{-2}, z = 2;$ *and* $\frac{x-1}{1} = \frac{2y+3}{3} = \frac{z+5}{2}$ (1)$\frac{\pi}{6}$ (2)$\frac{\pi}{4}$ (3)$\frac{\pi}{3}$ (4)$\frac{\pi}{2}$	**1**

<table>
<tr><td>8</td><td>The number given by the Rolle's theorem for the function $x^3 - 3x^2, x \in [0, 3]$ is
[1]1 [2]$\sqrt{2}$ [3]$\frac{3}{2}$ [4]2</td><td></td></tr>
<tr><td>9</td><td>If we measure the side of a cube to be 4 cm with an error of 0.1 cm, then the error in our calculation of the volume is
[1]0.4 cu. cm [2]0.45 cu.cm [3]2 cu. cm [4]4.8 cu. cm</td><td>1</td></tr>
<tr><td>10</td><td>$Find\ the\ value \int_0^{\frac{2}{3}} \frac{dx}{\sqrt{4-9x^2}}$
[1]$\frac{\pi}{6}$ [2]$\frac{\pi}{2}$ [3]$\frac{\pi}{4}$ [4]π</td><td>1</td></tr>
<tr><td>11</td><td>The integrating factor of the differential equation $\frac{dy}{dx} + P(x)y = Q(x)$ is x, then $P(x)$
[1]x [2]$\frac{x^2}{2}$ [3]$\frac{1}{x}$ [4]$\frac{1}{x^2}$</td><td>1</td></tr>
<tr><td>12</td><td>The probability mass function of a random variable is defined as
<table>
<tr><td>x</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td></tr>
<tr><td>$f(x)$</td><td>k</td><td>$2k$</td><td>$3k$</td><td>$4k$</td><td>$5k$</td></tr>
</table>
The $E(X)$ is equal to
[1]$\frac{1}{15}$ [2]$\frac{1}{10}$ [3]$\frac{1}{3}$ [4]$\frac{2}{3}$</td><td>1</td></tr>
<tr><td>13</td><td>Subtraction is not a binary operation in
(1)R (2) Z (3) N (4) Q</td><td>1</td></tr>
<tr><td>14</td><td>The domain of the function defined by $f(x) = \sqrt{x-1}$ is
(1)$[1, 2]$ (2)$[-1, 1]$ (3)$[0, 1]$ (4)$[-1, 0]$</td><td>1</td></tr>
<tr><td>15</td><td>Find the general equation of a circle with centre $(-3, -4)$ and radius 3units.
(1)$x^2 + y^2 + 6x + 8y + 6 = 0$ (2)$x^2 + y^2 + 6x + 8y + 16 = 0$
(3)$x^2 + y^2 = 0$ (4)$x^2 + y^2 + 16 = 0$</td><td>1</td></tr>
<tr><td>16</td><td>$If\ \ 2\hat{\imath} - \hat{\jmath} + 3\hat{k}, 3\hat{\imath} + 2\hat{\jmath} + \hat{k}, \hat{\imath} + m\hat{\jmath} + 4\hat{k}\ are\ coplanar, find\ the\ value\ of\ \ m$
[1]3 [2]-3 [3]$\frac{3}{2}$ [4]0</td><td>1</td></tr>
<tr><td>17</td><td>Evaluate: $x \to 0 \left(\frac{sinsin\ mx}{x}\right)$.
[1]0 [2]$-m$ [3]m [4]∞</td><td>1</td></tr>
</table>

18	$g(x) = x^2 + sin\, sin\, x$ the find dg. $[1] dg = (2x + cosx)dx$ $[2] dg = (2x - cosx)dx$ $[3] dg = (x + 2cosx)dx$ $[4] dg = (2x + sinx)dx$	**1**
19	$Find\ order\ and\ degree\ \frac{d^3y}{dx^3} = (xy)^2 + \left(\frac{d^2y}{dx^2}\right)$ [1] 3,0 [2]3,1 [3]2,1 [4]2,2	**1**
20	$z = 5 - 2i$ and; $w = -1 + 3i\ then\ find\ \ z + w$ [1] $4 + i$ [2]$-4 + i$ [3]]$-6 + 5i$ [4]]$-5 + 6i$	**1**

PART II .

Answer any seven questions. question number 30 is Compulsory . 7 X 2 = 14

21	$If\ adj\ A = [-1\ 2\ 2\ 1\ 1\ 2\ 2\ 2\ 1\ ], find\ A^{-1}$.	**2**
22	$Show\ that\ \ Z = \left(2 + i\sqrt{3}\right)^{10} + \left(2 - i\sqrt{3}\right)^{10} is\ real$	**2**
23	Find the monic polynomial equation of minimum degree with real coefficients having $2 - \sqrt{3}i$ as a root.	**2**
24	Identify the type of conic sections for the equations $y^2 + 4x + 3y - 4 = 0$	**2**
25	Find df for $f(x) = x^2 + 3x$ and evaluate it for $x = 2$ and $dx = 0.1$	**2**
26	$y = Ae^{12x} + Be^{-12x}$ Show that the differential equation $is\ \frac{d^2y}{dx^2} - 144y = 0$ corresponding to the family of curves represented by the equation where A and B are arbitrary constants	**2**
27	$Find\ the\ slope\ of\ the\ tangent\ to\ the\ curves\ at\ the\ respective\ given\ points.$ $y = x^4 + 2x^2 - x$ at $x = 1$	**2**
28	$Evaluate: \int_0^3 (3x^2 - 4x + 5)dx$	**2**
29	Suppose X is the number of tails occurred when three fair coins are tossed once simultaneously. Find the values of the random variable X and number of points in its inverse images.	**2**
30	Find the length of the perpendicular from the point $(1, -2, 3)$ to the plane $x - y + z = 5$	**2**

PART III .

Answer any seven questions. question number 40 is Compulsory . **7 X 3 = 21**

31	Show that the rank matrix:$[\ 3\ \ -8\,5\ \ \ 2\ \ \ 2\ \ -5\,1\ \ \ 4\ -1\ \ \ 2\,3\ \ -2\ ]$; is 3.	**3**
32	The complex numbers u, v, and w are related by $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$. If $v = 3 - 4i$ and $w = 4 + 3i$, find u in rectangular form.	**3**
33	Find the value of $\left(cos\,cos\,\left(\frac{4\pi}{3}\right)\right) + \left(cos\,cos\,\left(\frac{5\pi}{4}\right)\ \right)$	**3**
34	$If\ p\ and\ q\ are\ the\ roots\ of\ the\ equation\ lx^2 + nx + n = 0,$ $show\ that\ \sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0.$	**3**
35	$U(x, y, z) = xyz, x = e^{-t}, y = e^t\,cos\,cos\,t\ , z = sin\,sin\,t\ , t \in R.\ ; then\ find\ \frac{dU}{dt}$	**3**
36	Find the volume of a right-circular cone of base radius *r* and height *h*.	**3**
37	Construct the truth table $(\ p \vee q) \vee \neg q$.	**3**
38	If the equation $3x^2 + (3 - p)xy + qy^2 - 2px = 8pq$ represents a circle, find p and q. Also determine the centre and radius of the circle.	**3**
39	If $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}, \vec{b} = 2\hat{\imath} - \hat{\jmath} + \hat{k}, \vec{c} = 3\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{a} \times (\vec{b} \times \vec{c}) = l\vec{a} + m\vec{b} + n\vec{c}$, find the values of l, m, n.	**3**
40	Solve $(1 + x^2)\frac{dy}{dx} = 1 + y^2$.	**3**

PART IV

Answer all the questions . **7 X 5 = 35**

41	**(a)**Investigate for what values of λ and μ the system of linear equations x + y + 3z = 0,4x + 3y + μz = 0, 2x + y + 2z = 0 has (i) trivial solution, (ii) non-trivial solution. **(OR)** **b)** Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 metre high?	5

42	**a)**If $z = x + iy$ and arg $\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$, show that $x^2 + y^2 = 1$. **(OR)** $b\ )\ Solve\ the\ equation\ 6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0\ if\ it\ is\ known\ that\ \frac{1}{3}\ is\ a\ solution.$	
43	(a) A closed (cuboid) box with a square base is to have a volume of 2000 c.c. The material for the top and bottom of the box is to cost Rs. 3 per square cm and the material for the sides is to cost Rs. 1.50 per square cm. If the cost of the materials is to be the least, find the dimensions of the box. **(OR)** **b)** The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that the number triples in 5 hours, find how many bacteria will be present after 10 hours?	5
44	$a)$ Prove by vector method that $sin\ sin\ (\ \alpha - \beta) = sin\ sin\ \alpha\ \ cos\ cos\ \beta\ \ - cos\ cos\ \alpha\ \ sin\ sin\ \beta$. **(OR)** **b)** A rectangular page is to contain $24cm^2$ of print. The margins at the top and bottom of the page are 1.5 cm and the margins at other sides of the page is 1 cm. What should be the dimensions of the page so that the area of the paper used is minimum?	5
45	a) Find the parametric form of vector equation, and Cartesian equations of the plane passing through the points $(2,2,1), (9,3,6)$ and perpendicular to the plane $2x + 6y + 6z = 9$.. **(OR)** **b)** Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation $+_5$ on Z_5 using table corresponding to addition modulo 5.	5
46	**a)**. Father of a family wishes to divide his square field bounded by $x = 0$, $x = 4$, $y = 4$ and $y = 0$ along the curve $y^2 = 4x$ and $x^2 = 4y$ into three equal parts for his wife, daughter and son. Is it possible to divide? If so, find the area to be divided among them. **(OR)** b**)**. A six sided die is marked '1' on one face, '3' on two of its faces, and '5' on remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find (i) the probability mass function (ii)the cumulative distribution function (iii) $P(4 \leq X \leq 10)$ (iv) $P(X \geq 6)$	
47	**a)** if $u = sec^{-1}\left[\frac{x^3-y^3}{x+y}\right]$ then prove that; $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\ cot\ cot\ u$. **(OR)** b) Solve the following differential equations: $(x^3 + y^3)dy - x^2ydx = 0$.	5